Test 1 Computational Methods of Science/Computational Mechanics, December 2019

Duration: 2 hours.
In front of the questions, one finds the points. The sum of the points plus 1 gives the end mark for this test. Criteria used for the grading are: demonstration of understanding, logical reasoning, correct use of terminology, correctness of results.

Consider on (0,1) the differential equation

$$-\frac{d}{dx}(\exp(x)\frac{du}{dx}) + \frac{1}{1+x}u = x$$

with boundary condition sets:

1. $u(0) = u(1) = 0$,
2. $u(0) = 8$ and $u(1) = -3$,
3. $u(0) = 0$ and $\frac{du}{dx}(1) + u(1) = 2$.

1. [1.5] Derive the weak (Galerkin) form and the associated function space of this equation for boundary conditions set 1. Give also the bilinear and linear form, where in the bilinear form only first derivatives appear.

2. [1.0] Show that the bilinear form occurring in the previous part is positive definite.

3. [1.5] Given the interpolation points $x_i = ih$, with $h = 1/n$ on which we define the space of piecewise linear interpolation polynomials V_h in which each element u_h has the property $u_h(0) = u_h(1) = 0$. Determine the stiffness matrix and the load vector resulting from the weak problem on this subspace. It is enough to finally give the integrals which have to be determined.

4. [1.0] Show that the matrix determined in the previous part is symmetric and positive definite.

5. [0.5] Give the associated minimization problem. Also here, express your final results in integrals.

6. [1.5] What changes in the weak form and function space if we replace boundary conditions set 1 by set 2?

7. [1.7] What changes in the weak form and function space if we replace boundary conditions set 1 by set 3?

8. [0.3] Suppose we use instead of piecewise linear polynomial interpolation piecewise cubic polynomial interpolation. What will be the expected order of convergence of $||u - u_h||$ and of $||u - u_h||_{H_1}$, respectively? Here u is the exact solution and u_h the solution on the subspace.